Chapter 3 – Significant Figures

Overview

“Significant figures” is a term that refers to the number of digits in an experimentally derived number that give useful information about the data quality. Data with many significant figures is considered to be precise, and usually implies greater accuracy. In this chapter, students will learn the rules for writing and manipulating significant digits. They will use this knowledge to give the correct number of significant digits for data collected in this lab.

Teaching About Significant Figures

Everybody has problems with significant figures! It’s difficult to figure out why we need significant figures, what the rules are for finding out how many significant figures a number has, and how to do mathematical operations with significant figures.

The single most important thing to teach about significant figures is that we need them because we live in the real world. Although we can imagine finding a measurement to perfect accuracy with some hypothetical instrument, we never actually do because real instruments aren’t infinitely accurate. Because our instruments aren’t perfect, it’s important that we somehow indicate how good our instruments are to anybody looking at our data. We do this by limiting the number of digits we write in a measured number to the significant figures. An example, if I were to tell you that I weighed 80.6388 kilograms, you’d probably assume that I gave you four digits past the decimal because I weighed myself on a special scale that can measure things to that degree of precision. You wouldn’t assume I just used my bathroom scale because the number of significant figures is too high.

Another problem everybody has is with the difference between precision and accuracy. Precision is a measure of how reproducably you can take a measurement. For example, if you measure the weight of a paper clip to be 1.0025 grams, 1.0026 grams, and 1.0025 grams when you weigh it three times, you have a very precise measurement. High precision (or high reproducability) is denoted by a large number of significant figures.

Accuracy, on the other hand, is a measure of how close a measured value is to the actual value. If the paper clip I weighed in the example above actually weighed 1.9871 grams, my measurements wouldn’t have been very accurate because they don’t reflect the true mass of the paper clip.
As you might imagine, very precise measurements are usually also very accurate. This runs counter to what chemistry textbooks say, which is that precision and accuracy are independent of one another. Imagine this: You’re designing an instrument that’s supposed to reproducibly measure the weight of very small objects to a very high degree of precision. When you’re done, the balance can weigh objects accurately to the nearest millionth of a gram. If you went to all the time and trouble to make such an instrument, wouldn’t you also spend the time and trouble to make that instrument accurate?

Typically, very high quality instruments measure things with high precision and accuracy. High quality instruments that are out of calibration are able to measure things with high precision but low accuracy. Inexpensive instruments usually have low precision and accuracy. That’s why a laboratory balance costs $300 USD while a bathroom scale costs $8.

When you’re teaching your students the rules for significant figures [p. X], make sure you give your students several examples of each. There’s only so far you can go with teaching the definitions – real comprehension comes with repetition. The worksheets included with this lab are good introductions to significant figure problems, but you’ll probably want more examples to do as homework or in-class assignments. The more that students practice significant figure problems, the better they’ll be at doing them.

### Doing the Significant Figure Lab

**Equipment:**

**Station 1:**
- One 50 mL graduated cylinder
- Three disposable pipets
- Three pen caps (large marker caps work best)
- 100 mL tap water in a beaker

**Station 2:**
- One 10 mL graduated cylinder (must have 0.1 mL gradations)
- Three disposable pipets (should be identical to station 1)
- Three pen caps (should be identical to station 1)
- 100 mL tap water in a beaker

**Station 3:**
- Five pennies
- Five large paper clips
- One digital balance
Station 4:
- Five pennies
- Five large paper clips (identical to station 3)
- One triple-beam balance

Stations 5 and 6:
- One pad of Post-it™ notes at each station
- Five large paper clips at each station

The stations in this lab are paired: Stations 1 and 2 are paired, as are 3 to 4 and 5 to 6. All equipment in the paired stations should be identical. For example, the paper clips in stations 5 and 6 should be from the same box.

Safety:
There are no safety issues in this lab and goggles are not required.

Room destruction factor:
Because the equipment used doesn’t generate a mess, neither will this lab.

How the lab works:
Each pair of stations is measuring the same thing: Stations 1 and 2 measure volume, stations 3 and 4 measure mass, and stations 5 and 6 measure length. Ideally, the measurements should be the same, varying only in the number of significant figures that are appropriate for each measuring tool. As a result, most of the grading for this lab is not in the absolute value of the measurements but in the number of decimal places used to write the measurements. Make sure your students are clear on how to properly write the correct number of significant figures for a measured value before setting them loose.

This lab goes quickly. Three to five minutes for each lab station should be sufficient time to complete the questions before rotating to the next station.

What can go wrong:
- It may not be a bad idea to review the use of a triple-beam balance before starting this lab. Some students will need the tutorial before starting station 4.
- The objects being measured in this lab frequently get lost. The equipment list takes this into account, but be ready to produce more replacements if you have a lot of classes doing this lab at once.
Solutions for the Significant Figure Lab

Stations:

Station 1: Because 50 mL graduated cylinders measure volume to the nearest milliliter, answers should be given to the nearest 0.1 mL.

Station 2: Because the 10 mL graduated cylinder will measure volume to the nearest 0.1 mL, answers should be given to the nearest 0.01 mL.

Station 3: The number of significant figures given in each response should be the same as the number of significant figures on the electronic balance readout. Make sure students are not estimating an extra digit, as they do in the other stations.

Station 4: Student answers should have one more decimal place than the smallest gradations on the triple beam balance.

Station 5: Student answers should be to the nearest tenth of a centimeter, as the ruler given to them accurately measures centimeters.

Station 6: Student answers should be to the nearest hundredth of a centimeter, as their ruler measures to the nearest millimeter.

Post-lab questions:

1) We cannot use as many significant figures as we want because significant figures are a reflection of the precision of the instrument used to take a measurement. It doesn’t make sense to measure down to the nearest millionth of a centimeter with an ordinary ruler because the ruler simply isn’t that sensitive – any digits past the first estimated one are random guesses.

2) If we had an accurate enough instrument, there would be no problem with writing fifteen decimal places. However, for this to be valid, the measurement tool would have to have gradations accurate to 14 decimal places so we could estimate the fifteenth digit.

3) Precision is a measure of how reproducible a measurement is and is denoted by the number of significant figures written in a number. Accuracy is a measure of how close a measured value is to the actual value. Though precision and accuracy aren’t necessarily related, precise measurements are frequently also very accurate.

Solutions for the “Significant Figures Worksheet”

1) three 8) two
2) three 9) three
3) three 10) four
4) two 11) three
5) four 12) three
6) one 13) two
7) four 14) three (continued next page)
15) A precise measurement would not be accurate if a very precise piece of uncalibrated equipment was used. In such a case, there would be the same systematic error in each measurement.

16) Lack of precision implies that you can't reliably reproduce a measurement. If a measurement can't be reproduced, than many of the measurements around it will be inaccurate.

17) The ruler that measured “7.50 centimeters” was telling me that I am able to accurately measure the length of my thumb to the nearest hundredth of a centimeter. The ruler that measured “7.5 centimeters” only gives reliable information to the nearest tenth of a centimeter. The number is the same for both measurements, but the meaning of the number is not.

**Solutions for “Using Significant Figures in Calculations”**

1) \(6.84 \rightarrow 6.8\)
2) \(-0.5 \rightarrow -1\)
3) \(9.411 \rightarrow 9.41\)
4) \(7.888 \rightarrow 7.9\)
5) \(3.378261 \rightarrow 3.4\)
6) \(0.321488 \rightarrow 0.321\)
7) \(28080 \rightarrow 28,000 \text{ or } 2.8 \times 10^4\)
8) \(0.024 \rightarrow 0.02\)
9) \(78.512 \rightarrow 78.5\)
10) \((30.) \times (11.3) = 339 \rightarrow 340\)
11) \(7.451613 \rightarrow 7.5\)
12) \(65.0023 \rightarrow 65\)
13) \(3,610,349 \rightarrow 3,600,000 \text{ or } 3.6 \times 10^6\)
14) \((3.81) + 2.45001 – (920) = 913.74 \rightarrow 910\)
15) It’s important to use the correct number of significant figures when solving a problem because the number of significant figures in the answer tells the reader about how trustworthy the answer is. If you write too many significant figures, the data appears more precise than it really is. If you write too few significant figures, the data gives less useful information than it should.
Significant Figures

Significant figures are the digits in a measured number that indicate the measuring equipments degree of precision. Generally, when writing down a measurement, you should write all of the digits that you obtained directly with the measuring device and add a final digit that you’ve estimated. For example, if you have a ruler that can measure length in millimeters, you should write the lengths of objects you’ve measured to tenths of millimeters.

Rules for Writing Significant Figures:

1) **All nonzero digits are significant.** For example, “3.4 grams” has two significant figures.

2) **Zeros that are between nonzero digits are significant.** For example, “3.04 grams” has three significant figures.

3) **Zeros written to the left of all nonzero digits are not significant.** For example, “0.0034 grams” has two significant figures.

4) **Zeros written to the right of all nonzero digits are only significant if a decimal point is written in the number.** For example, “1000 grams” has one significant figure, while “1000.0 grams” has five. The zeros in the second number indicate that a value can be measured accurately to the nearest tenth of a gram, while writing simply “1000 grams” indicates that the measurement has been rounded to the nearest thousand grams. While both mean the same thing to your calculator, they don’t mean the same thing to a reader.

5) **Numbers in scientific notation have the same number of significant figures as the portion of the number that’s before the “x 10^n” part of the number.** For example, “4.30 x 10^5 grams” has three significant figures.

Rules for Using Significant Figures in Calculations:

1) **When adding or subtracting, the answer should have the same number of figures to the right of the decimal as the value with the fewest decimal places.** For example, 3.4 + 5.023 = 8.423 → Round this to 8.4, because 3.4 has only one digit to the right of the decimal.

2) **When multiplying or dividing, the answer should have the same number of significant figures as the value with the fewest significant figures.** For example, 1.220 x 3.4870 = 4.25414 → Round this answer to 4.254, because 1.220 has only four significant figures.
Significant Figures Lab

In chemistry, we try to get the most information we can out of every measurement. Whenever we write down a number we’ve measured, the number of digit the number has reflects the precision of the instrument we used to get it.

This is important in science because when we read somebody else’s data we like to know how precise their data really is. If we use the wrong number of digits in our answers, we might fool people into believing that imprecise data is really precise, or vice versa.

When taking measurements, you should always write all values so they show the smallest marking on the instrument, plus an extra digit that you estimate. For example, if you use a ruler that has lines for millimeters, you should write your answers to the nearest tenth of a millimeter because you can estimate the last digit. The exception to this rule is digital equipment, such as an electronic balance. Because you can’t estimate the last digit on a digital balance, simply write down the answer on the readout.

In this lab, you will be measuring length, volume, and mass using common laboratory instruments. For each of these tools, you must write down your answer with the correct number of significant digits! Remember, the number of digits you write depends on the instrument you used to take the measurement.

Station 1: Measuring volume with a 50 mL graduated cylinder

Use the 50 milliliter graduated cylinder at this station to find the following volumes. Be sure to use the proper number of significant figures in your answer!

1) What is the maximum volume of the pipet? ____________
2) What is the maximum volume of the pen cap? ____________

Station 2: Measuring volume with a 10 mL graduated cylinder

Use the 10 milliliter graduated cylinder at this station to find the following volumes. Be sure to use the proper number of significant figures in your answer!

1) What is the maximum volume of the pipet? ____________
2) What is the maximum volume of the pen cap? ____________
Station 3: Measuring mass with an electronic balance

Use the electronic balance at this station to find the following weights. Be sure to use the proper number of significant figures in your answer!

1) What is the mass of the penny? _______________

2) What is the mass of the paper clip? _______________

Station 4: Measuring mass with a triple beam balance

Use the triple beam balance at this station to find the following weights. Be sure to use the proper number of significant figures in your answer!

1) What is the mass of the penny? _______________

2) What is the mass of the paper clip? _______________

Station 5: Measuring distance with a ruler

Using the ruler printed below, find the following lengths. Be sure to use the proper number of significant figures in your answer!

Ruler:

<table>
<thead>
<tr>
<th>0</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>centimeters</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1) What is the length of the post-it note? _______________

2) What is the length of the paper clip? _______________

Station 6: Measuring distance with a ruler

Using the ruler printed below, find the following lengths. Be sure to use the proper number of significant figures in your answer!

Ruler:

<table>
<thead>
<tr>
<th>0</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>centimeters</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1) What is the length of the post-it note? _______________

2) What is the length of the paper clip? _______________
Post-lab questions:

1) Why can’t we write numbers with as many significant figures as we want? For example, if we measure something with an ordinary ruler, why is it wrong to write our measurement as “0.928772662 centimeters”? Explain.

2) If we had an accurate enough instrument, is there any reason we couldn’t write down a value to 15 decimal places (as in the number 0.123456789012345)? Explain.

3) In your own words, what’s the difference between precision and accuracy?
Significant Figures Worksheet

For problems #1-14, write down how many significant figures each number has:

1) 4.53 __________
2) 2.30 __________
3) 1.02 __________
4) 4500 __________
5) 3200. __________
6) 0.002 __________
7) 1.000 __________
8) 0.070 __________
9) 0.707 __________
10) 7.070 __________
11) 7070 __________
12) 0.700 __________
13) 3.4 \times 10^4 __________
14) 1.02 \times 10^2 __________

Please answer the following questions:

15) Under what circumstances might a very precise measurement not be accurate? Explain.

16) Are there any circumstances under which an accurate measurement may not be precise? Explain.

17) I measured the length of my thumb and found that it is 7.50 centimeters long. When I used another ruler, I found that the length was 7.5 centimeters. Explain the difference between these two measurements.
Using Significant Figures in Calculations

Solve each of the following math problems and write their answers with the correct number of significant figures:

1) \(4.5 + 2.34 = \) _________________________
2) \(4.5 - 5 = \) _________________________
3) \(6.00 + 3.411 = \) _________________________
4) \(3.4 \times 2.32 = \) _________________________
5) \(7.77 / 2.3 = \) _________________________
6) \(3.890 / 121 = \) _________________________
7) \(1200 \times 23.4 = \) _________________________
8) \(120 \times 0.002 = \) _________________________
9) \(78.5 + 0.0021 + 0.0099 = \) _________________________
10) \((3.4 \times 8.90) \times (2.3 + 9.002) = \) _________________________
11) \((2.31 \times 10^3) / (3.1 \times 10^2) = \) _________________________
12) \(0.0023 + 65 = \) _________________________

Advanced problems:

13) \((3.4 \times 10^5) + 210,349 = \) _________________________
14) \(1.09 \times 3.498 + 2.45001 - 2.123 / 0.0023 = \) _________________________

15) Why is it important to always use the correct number of significant figures when solving a problem?